Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

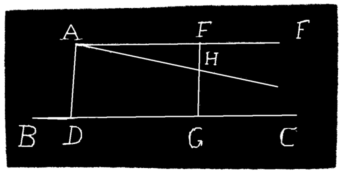
Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

to a given line. Draw AD perpendicular to BC, and through A draw AE perpendicular to AD. Then (1st and 2d) AE and DC are parallel; and the perpendicular FG equals AD, by (3d). Now suppose another line AH parallel to BC. Then HG equals AD or its equal FG. When HG equals FG, AH and AF coincide. Therefore, through a given point one line, and only one, can be drawn parallel to a given line.



The above demonstration may be made without the use of the word parallel. Thus: Through a given point one line, and only one, can be drawn equidistant from a given line.

With the figure drawn as in No. 3, begin the demonstration with the words: From *any* point F let fall the perpendicular &c., to prove the lines equidistant. Then with the same figure as in No. 4, and substituting the word equidistant for parallel, we have the demonstration.

CRADLE-ROCKING BY ELLEPTIC FUNCTIONS.

By F. P. MATZ, M. Sc., Ph. D., New Windsor, Maryland.

After adopting the *gravitation-unit* of force, the equation of motion of the pendulum may be written $(h^2+k^2)(W/g)(d^2\theta/dt^2)=-Wh\sin\theta\ldots.(1)$. Briefly making $(h+k^2/h)=l$ and $g/l=n^2$, we obtain from (1)

$\frac{1}{2}(d\theta/dt)^2=n^2(\text{vers}\,\alpha-\text{vers}\,\theta)\ldots.(2)$, in which, according to Sir William Thomson (Lord Kelvin), n is the *angular speed* of the pendulum, Divide *semicircularly* the pendulum-bob, turn downward the convex sides of these divisions centrally joined by a rectilinear axis of inappreciable length, and let the pendulum-rod bisect this rectilinear axis. In the position specified. these divisions constitute the *rockers* of an old-fashioned cradle; and this cradle we regard as placed upon a perfectly rough horizontal plane. Detaching the pendulum rod from the point of suspension, we have to consider *the rocking*, or the rolling oscillations on a horizontal plane, of a material body resting on a semicircular base. Let r=the radius of the equal semicircular rockers. Consider the *line* joining the points of tangency of the rockers with the horizontal plane, as the instantaneous axis of rotation; then, after obvious transformations, (2) becomes $\frac{1}{2}(r^2-2hr\cos\theta+h^2+k^2)(d\theta/dt)^2=gh(\text{vers}\,\alpha-\text{vers}\,\theta)\ldots.(3)$.

$$\therefore \left(\frac{dt}{d\theta}\right)^2 = \frac{[(r-h)^2+k^2]\cos^2\tfrac{1}{2}\theta+[(r+h)^2+k^2]\sin^2\tfrac{1}{2}\theta}{4gh(\sin^2\tfrac{1}{2}\alpha-\sin^2\tfrac{1}{2}\theta)},$$

$$= \frac{[(r-h)^2+k^2]\cos^2\tfrac{1}{2}\theta+[(r+h)^2+k^2]\sin^2\tfrac{1}{2}\theta}{4gh[\sin^2\tfrac{1}{2}\alpha-(1-\sin^2\tfrac{1}{2}\alpha)\tan^2\tfrac{1}{2}\theta]},$$

$$= \frac{[(r-h)^2+k^2]+[(r+h)^2+k^2]\tan^2\tfrac{1}{2}\theta}{4gh\cos^2\tfrac{1}{2}\alpha(\tan^2\tfrac{1}{2}\alpha-\tan^2\tfrac{1}{2}\theta)} \dots . (4).$$

In order to transform (4), put $\tan\frac{1}{2}\theta=\tan\frac{1}{2}\alpha\cos\phi$; then differentiating,

etc., $\left(\dfrac{d\theta}{d\phi}\right)^2 = 4\left(\dfrac{\tan\frac{1}{2}\alpha\sin\phi}{1+\tan^2\frac{1}{2}\alpha\cos^2\phi}\right)^2, = 4\left(\dfrac{\sin\frac{1}{2}\alpha\cos\frac{1}{2}\alpha\sin\phi}{1-\sin^2\frac{1}{2}\alpha\sin^2\phi}\right)^2 \dots . (a).$

$$\therefore \left(\frac{dt}{d\phi}\right)^2 = \frac{[(r-h)^2+k^2]\cos^2\tfrac{1}{2}\alpha+(1-\sin^2\phi)[(r+h)^2+k^2]\sin^2\tfrac{1}{2}\alpha}{gh(1-\sin^2\tfrac{1}{2}\alpha\sin^2\phi)^2} \dots . (5).$$

Put $\varkappa^2 = \dfrac{[(r+h)^2+k^2]\sin^2\frac{1}{2}\alpha}{[(r+h)^2+k^2]\sin^2\frac{1}{2}\alpha+[(r-h)^2+k^2]\cos^2\frac{1}{2}\alpha} \dots . (b),$

and $\varkappa'^2 = \dfrac{[(r-h)^2+k^2]\cos^2\frac{1}{2}\alpha}{[(r+h)^2+k^2]\sin^2\frac{1}{2}\alpha+[(r-h)^2+k^2]\cos^2\frac{1}{2}\alpha} \dots . (c);$

then will $\varkappa^2+\varkappa'^2=1$. Make $n=\sqrt{(g/r)}$, and represent the denominators of (*b*) and (*c*) by M; then (5) may be written

$$ndt = \sqrt{\left(\frac{M}{hr}\right)}\left[\frac{(1-\varkappa^2\sin^2\phi)d\phi}{(1-\sin^2\tfrac{1}{2}\alpha\sin^2\phi)\sqrt{(1-\varkappa^2\sin^2\phi)}}\right] \dots . (6).$$

According to the Jacobian system of notation as modified by Gudermann (*Theorie der Modular Functionen*), we have $\phi=\mathrm{am}\,U$, and $\sin\frac{1}{2}\alpha=\varkappa\,\mathrm{sn}\,A$.

Since $\mathrm{dn}^2A+\varkappa^2\mathrm{sn}^2A=1$, we obtain $\mathrm{dn}^2A=1-\varkappa^2\mathrm{sn}^2A=1-\varkappa^2(\sin^2\frac{1}{2}\alpha/\varkappa^2)=\cos^2\frac{1}{2}\alpha$; also,

$$\mathrm{sn}\,A = \sqrt{\left(\frac{[(r+h)^2+k^2]\sin^2\tfrac{1}{2}\alpha+[(r-h)^2+k^2]\cos^2\tfrac{1}{2}\alpha}{[(r+h)^2+k^2]}\right)} \dots . (d),$$

and $\mathrm{cn}\,A=\sqrt{(1-\mathrm{sn}^2A)}, = \dfrac{2\cos\frac{1}{2}\alpha\sqrt{(hr)}}{\sqrt{[(r+h)^2+k^2]}} \dots . (e).$

$$\therefore\ ndt = 2\left(\frac{\mathrm{sn}A\,\mathrm{dn}A}{\mathrm{cn}A}\right)\left[1-\frac{\varkappa^2\mathrm{cn}^2A\,\mathrm{sn}^2U}{1-\varkappa^2\mathrm{sn}^2A\,\mathrm{sn}^3U}\right]dU \dots . (7),$$

and $nt=2[(\mathrm{sn}A\,\mathrm{dn}A/\mathrm{cn}A)\,U-\Pi(U,A)] \dots . (8)$, while $\tan\frac{1}{2}\theta=\tan\frac{1}{2}\alpha\,\mathrm{cn}\,U$.